Developments in Time Analysis of Particle and Photon Tunnelling

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Abstract

A new systematization of various theoretical approaches to defining tunnelling times for nonrelativistic particles in the light of time as a quantum–mechanical observable is given. Then new results on the analogy between particle and photon tunnelling and on time as an observable in quantum electrodynamics and also analysis of the causality validity during tunneling are presented.

I. INTRODUCTION

Tunnelling time analysis has a long history. The problem of defining the tunnelling time was posed already in the beginning of the 30–th [1,2]. But from then it was remained almost ignored until the 50–60–th when the more general problem of defining the quantum–collision duration began to be investigated [3–14] and almost simultaneously, after a long period of the silence from the 20–th induced by the Pauli theorem about the impossibility to construct

the self–adjoint time operator in quantum mechanics [15], the first attempts appeared to introduce the notion of time as a quantum–mechanical observable [16–22]. And during the 70–80–th (mainly in [23–25]) the membership of time in the set of quantum–mechanical observables had been principally cleared up. The detailed analysis of developments in the study of time as a quantum–physical observable is contained in [25–27].

The developments in physics of condensed media, physics of electromagnetic–wave propagation, biophysics and especially the advent of high–speed electronic devices, based on tunnelling processes, revived an interest in the tunnelling time analysis, whose relevance has always been apparent in physics of nuclear sub–barrier fission and fusion, and stimulated the publication of not only a lot of theoretical studies but already a number of theoretical reviews [28–36].

Regarding the experimental research of tunnelling times, the great difficulties of real measurements for particles (in particular, too small values of tunnelling times) made the verifications of theoretical results to be practically impossible (see, for instance, [37,38]). Only recently there were realized some measurements of tunnelling times for microwaves and laser–light photons [39–42]. In what sense the results of these experiments are suitable for the time analysis of particle tunnelling ? Although the formal analogy of particle and photon (electromagnetic wave–packet) tunnelling is well seen by simple comparing the relevant stationary equations [43–46], actually we deal with the *time–dependent* equations and moreover, the problem of time as an observable also in quantum electrodynamics must be resolved too. Below (in Sec.7) these questions will be explicitly analyzed.

Returning to the problem of the theoretical definition of the tunnelling time for particles, we see not only the absence of the consensus in such definition but also declarations about the incompatibility of some approaches both quantitatively and in the physical interpretation [28–36]. Among the reasons of such situation there are the following ones: *(i)* the problem of defining the tunnelling time is closely connected with general fundamental problems of time as a *quantum–physical observable* and the general definition of quantum–collision durations. And the acquaintance with the principal solution of these problems have

not got a wide prevalence yet. *(ii)* The motion of particles inside a potential barrier is a quantum phenomenon *without* any direct classical limit. *(iii)* There are essential *differences* in *initial, boundary* and *external conditions* of various definition schemes which have not been systematically analyzed yet.

Following [36,47], we can arrange the majority of approaches into several groups which are based on: (1) the time–dependent wave–packet description; (2) averaging over an introduced set of kinematics paths, distribution of which is supposed to describe the particle motion inside the barrier; (3) introducing a new degree of freedom, constituting a physical clock for measurements of tunnelling times. Separately, by one's self, the dwell time stands. The last has *ab initio* the presumptive meaning of the time that the incident flux has to be turned on, to provide the accumulated particle storage in the barrier [5,36].

The first group contains the so–called phase times, firstly mentioned in [3,4] and applied to tunnelling in [48,49], the times of the motion of wave–packet spatial centroids, earlier considered for general quantum collisions in [13,14] and applied to tunnelling in [50,51], and finally the Olkhovsky–Recami (O–R) method [35,52,53] of averaging over unidirectional fluxes, basing on the representation of time as a quantum–mechanical observable and on the generalization of the definitions,introduced in [7,25,26] for atomic and nuclear collisions. The second group contains methods, utilizing the Feynman path integrals [54–57], the Wigner distribution paths [58,59] and the Bohm approach [60]. To the third group the approaches with the Larmor clock [61] and the oscillatory barrier [62,63] pertain.

Certainly the basic self–consistent definition of tunnelling durations (mean values, variances of distributions and so on) has to be elaborated quite *similarly* to the definitions of other physical quantities (distances, energies, momenta, etc.,) on the base of utilizing all necessary properties of time as a quantum–physical observable (time operator, canonically conjugated to energy operator; the equivalency of the averaged quantities in time and energy representations with adequate measures, or weights, of averaging). Since the representation of solutions of the time–dependent Schrödinger equation as moving wave packets is typical and the most self–consistent in quantum collision theory (see, for instance, [6, ref.3], it is

natural to apply the wave–packet description. Then one can hope that in the framework of the conventional quantum mechanics every known definition of tunnelling times can be shown, after appropriate analysis, to be (at least in the asymptotic region, used for typical boundary conditions in quantum collision theory) either a *particular* case of the general definition or an *equivalent* one or the definition which is valid *not* for tunnelling but for some accompanying process, different from tunnelling.

Here such a definition with the necessary formalism is presented (Sec.2) and, without claiming to present the exhaustive analysis of all known definitions, then analysis of various approaches is given (Sec.3–5), basing on the O–R formalism. In Sec.6 some peculiarities of tunnelling evolution revealed by use of the O–R method are presented, then, after Sec.7, there is a short note on the reshaping (and reconstructing) phenomenon in connection with a possible formulation of relativistic causality in the cases of superluminal effective tunnelling velocities (Sec.8) and finally in Sec.9 some conclusions and reasonings on nearest prospects are presented.

II. THE OLKHOVSKY–RECAMI FORMALISM OF DEFINING TUNNELLING DURATIONS, BASED ON UTILIZING PROPERTIES OF TIME AS A QUANTUM–MECHANICAL OBSERVABLE AND THE WAVE–PACKET DESCRIPTION.

We confine ourselves to the simplest case of particles moving only along the x–direction, and consider a time–independent barrier in the interval $(0, a)$; see Fig. 1, in which a larger interval (x_i, x_f), containing the barrier region, is also indicated. Following the definition of collision durations, put forth in [7,23,25,26] and generalized in [35,52,54], we can eventually define the mean values of the time at which a particle passes through position x, travelling in the positive or negative direction of the x–axis, and the variances of the distributions of these times, respectively, as

$$< t_{\pm}(x) > = \frac{\displaystyle\int_{-\infty}^{+\infty} t J_{\pm}(x,t)\,\mathrm{d}x}{\displaystyle\int_{-\infty}^{+\infty} J_{\pm}(x,t)\,\mathrm{d}x} \tag{1}$$

and

$$\mathrm{D}\, t_{\pm}(x) \;=\; \frac{\displaystyle\int_{-\infty}^{+\infty} t^2 J_{\pm}(x,t)\,\mathrm{d}x}{\displaystyle\int_{-\infty}^{+\infty} J_{\pm}(x,t)\,\mathrm{d}x} - [< t_{\pm}(x) >]^2 \tag{2}$$

where $J_{\pm}(x,t) = \mathrm{Re}[(i\hbar/m)(\Psi(x,t))(\partial\Psi^*(x,t)/\partial x)]$ being the probability flux density for an evolving wave packet $\Psi(x,t)$. We recall here the equivalence of time and energy representations, with appropriate measures of averaging, in the following sense: $< \ldots >_t = < \ldots >_E$ (index t is omitted in all expression for $< \ldots >_t$ for the sake of the simplicity). This equivalence is a consequence of the time–operator existence. So, we have a formalism for defining mean values, variances and other central moments related to the duration distributions of all possible kinds of collisions and interactions with arbitrary energies, including tunnelling. For instance, for transmissions from region I to region III, we have

$$< \tau_T(x_i, x_f) > = < t_+(x_f) > - < t_+(x_i) > \tag{3}$$

$$\mathrm{D}\tau_T(x_i, x_f) = \mathrm{D}t_+(x_f) - \mathrm{D}t_+(x_i) \tag{4}$$

with $-\infty < x_i \leq 0$ and $a \leq x_f < \infty$. For a pure tunnelling process one has

$$< \tau_{tun}(0, a) > = < t_+(a) > - < t_+(0) > \tag{5}$$

and

$$\mathrm{D}\tau_{tun}(0, a) = \mathrm{D}t_+(a) + \mathrm{D}t_+(0). \tag{6}$$

Similar expression we have for the penetration (into the barrier region II) temporal quantities $< \tau_{pen}(x_i, x_f) >$ and $\mathrm{D}\tau_{pen}(x_i, x_f)$ with $0 < x_f < a$. For reflections in any point $x_f \leq a$ one has

$$< \tau_R(x_i, x_f) > = < t_-(x_f) > - < t_+(x_i) > \tag{7}$$

and

$$\mathrm{D}\tau_R(x_i,x_f)=\mathrm{D}t_-(x_f)+\mathrm{D}t_+(x_i). \qquad (8)$$

We stress that these definitions hold within the framework of conventional quantum mechanics, *without* introducing any new physical postulate.

In the asymptotic cases, when $|x_i| >> a$,

$$<\tau_T^{as}(x_i,x_f)>=<t(x_f)>_T - <t(x_i)>_{in} \qquad (9)$$

and

$$<\tau_T^{as}(x_i,x_f)>=<\tau_T(x_i,x_f)>+<t_+(x_i)>-<t_+(x_i)>_{in} \qquad (10)$$

where $<\ldots>_T$ and $<\ldots>_{in}$ denote averagings over the fluxes corresponding to $\psi_T = A_T\exp(ikx)$ and $\psi_{in}=\exp(ikx)$ respectively.

For initial wave packets

$$\Psi_{in}(x,t)=\int_0^\infty G(k-\overline{k})\exp[ikx-iEt/\hbar)\mathrm{d}E$$

(where $E=\hbar^2k^2/2m; \int_0^\infty |G(k-\overline{k})|^2\mathrm{d}E=1; G(0)=G(\infty)=0; k>0$) with sufficiently small energy (momentum) spreads, when

$$\int_0^\infty v^n|GA_T|^2\mathrm{d}E \cong \int_0^\infty v^n|G|^2\mathrm{d}E \qquad n=0,1,$$

we get

$$<\tau_T^{as}(x_i,x_f)>\cong<\tau_T^{ph}(x_i,x_f)>_E \qquad (11)$$

where

$$<\ldots>_E=\frac{\int_0^\infty v|G(k-\overline{k})|^2\,\mathrm{d}E\ \ \{\ldots\}}{\int_0^\infty v|G(k-\overline{k})|^2\,\mathrm{d}E}$$

and

$$\tau_T^{ph}(x_i,x_f)=(1/v)(x_f-x_i)+\hbar d(\arg A_T)/dE \qquad (12)$$

is the phase transmission time obtained by the stationary–phase approximation. At the approximation when (11) is valid and with a small contribution of $Dt_+(x_i)$ into the variance $D\tau_T(x_i, x_f)$ (that can be realized for sufficiently large energy spreads, i.e., short wave packets), we get

$$D\tau_T(x_i, x_f) = \hbar^2 < (d|A_T|/dE)^2 >_E \Big/ < |A_T|^2 >_E \; . \tag{13}$$

For the opposite case of very small energy spreads (quasi–monochromatic particles) expression (13) becomes the item of $Dt_+(x_f)$ and $D\tau_T(x_i, x_f)$ which is born by the barrier influence.

When $|G|^2 \to \delta(E - \overline{E}), \overline{E}$ being $\hbar^2 \overline{k}^{\,2}/2m$, we get for $< \tau_T^{as}(x_i, x_f) > \cong < \tau_T^{ph}(x_i, x_f) >_E$ strictly the ordinary phase time, without averaging. For a rectangular barrier with height V_0 and $\chi a >> 1$ (where $\chi = [2m(V_0 - E)]^{1/2}/\hbar$), the expressions (11) and (13) for $x_i = 0$ and $x_f = a$ pass, in the same limit, into the known expressions

$$\tau_{tun}^{ph} = 2/v\chi \tag{14}$$

(see [48] and also [35,36]) and

$$(D\tau_{tun}^{ph})^{1/2} = ak/v\chi \tag{15}$$

(coincident with one of the Larmor times [61] and the Büttiker–Landauer time [63] and also the imaginary part of the complex time in the Feynman path–integration approach) respectively (see also [64]).

For the *real* weight amplitude $G(k - \overline{k})$, when $< t(0) >_{in} = 0$, from (10), we obtain

$$< \tau_{tun}(0, a) > = < t_{tun}^{ph} > - < t_+(0) > \; . \tag{16}$$

By the way, if the measurement conditions are chosen to be such that only the positive–momentum components of wave packets are registered, i.e., $\Lambda_{exp,+}\Psi(x_i, t) = \Psi_{in}(x_i, t); \Lambda_{exp,+}$ being the projector onto positive–momentum states, then for any x_i from $(-\infty, 0)$ and x_f from (a, ∞)

$$< \tau_T(x_i, x_f) >_{exp} = < t_T^{ph}(x_i, x_f) >_E \tag{17}$$

and

$$< \tau_{tun}(0, a) >_{exp} = < t_{tun}^{ph} >_E \tag{18}$$

because $< t(0) >_{exp} = < t(0) >_{in}$.

The main criticism, by authors of [36,51] and also [60,65], of any approach to the definition of tunnelling times, in which spatial or temporal averaging over moving wave packets is used, implies the lack of a causal relationship in evolution of an incoming peak or centroid, turning into an outgoing peak or centroid. It is clear already from the 60–th (see, for instance, [14]) that these reasonings are particularly true only for the spatial approach with finite (not asymptotic) distances from the interaction region. And this criticism concerns only the attempts of the authors of [51] to trace evolution of an incoming peak into an outgoing one but not, strictly speaking, the O–R definition of collision, tunnelling, transmission, penetration, reflection, durations, etc; our definition of the mean duration of any such process is, by no means, based on the assumption that the centroid (or peak) of the incident wave packet directly evolves into the centroid (or peak) of the transmitted and reflected packets, as it was erroneously claimed in [65], but does simply signify the difference between the mean time values of the passing of the final and initial wave packets through the appropriate points, *regardless of any intermediate motion, transformation and reshaping of these wave packets.* And for any collision (and so on) processes, as a *whole*, one can test the causality condition. However, there is no unique general formulation of the causality condition, necessary and sufficient for all possible cases of collisions (and not only for nonrelativistic wave packets, but also for relativistic ones). The simplest (or the most strong) nonrelativistic causality condition implies the non–negative values of the mean durations. However, this is a *sufficient* but *not the necessary* causality condition. Negative times (advance phenomena) were revealed even near nuclear resonances, distorted by the non–resonant background (see, in particular, [26]); similarly, advance phenomena can occur at the beginning of tunnelling (see Sec.6). Generally speaking, the complete causality condition can be connected not only with

the mean time duration but also with other temporal properties of the considered process. For example, the following variant of the causality condition seems to be somewhat more realistic: the difference between the *effective starts* of final and initial fluxes is non–negative, the effective start being defined as the difference between the mean instant of wave–packet passing through the appropriate point and the square root of the corresponding instant–distribution variance. And also this condition is only sufficient but not necessary because in many realistic cases wave packets have infinite and not very rapidly decreasing forward tails. Much more realistic formulations of the causality conditions for wave packets with infinite tails are presented in Sec.8.

III. THE MEANING OF THE MEAN DWELL TIME IN THE LIGHT OF THE OLKHOVSKY–RECAMI FORMALISM.

As it is known [66] (see also [52,53]) the mean dwell time can be presented in two equivalent forms:

$$< \tau^{dw}(x_i, x_f) >= \left[\int_{-\infty}^{\infty} \mathrm{d}t \int_{x_1}^{x_2} |\Psi(x,t)|^2 \mathrm{d}x\right] \left[\int_{-\infty}^{\infty} J_{in}(x_i,t)\mathrm{d}t\right]^{-1} \tag{19}$$

and

$$< \tau^{dw}(x_i, x_f) >= \left[\int_{-\infty}^{\infty} tJ(x_f,t)\mathrm{d}t - \int_{-\infty}^{\infty} tJ(x_i,t)\mathrm{d}t\right] \left[\int_{-\infty}^{\infty} J_{in}(x_i,t)\mathrm{d}t\right]^{-1} \tag{20}$$

with $-\infty < x_i \leq 0; a \leq x_f < \infty$. In its primary definition (19) another, than in Sec.2, measure (weight) was used in integrating over t. What is that measure and what is its meaning ? Taking into account equation $\int J_{in}(x_i,t)\mathrm{d}t = \int |\Psi(x,t)|^2 \mathrm{d}x$, which follows from the continuity equation, one can easily see that this measure is $dP(x,t) = |\Psi(x,t)|^2 dx / \int |\Psi(x,t)|^2 \mathrm{d}x$ and it has the well–known quantum–mechanical meaning of the probability for a particle to be *found (localized)* or to *stay (dwell)*, in the spatial region $(x, x + \mathrm{d}x)$ at the moment t, independently on the motion processes. Then the quantity $P(x_1, x_2; t) = \int_{x_1}^{x_2} |\Psi(x,t)|^2 \mathrm{d}x / \int_{-\infty}^{\infty} |\Psi(x,t)|^2 \mathrm{d}x$ has the evident meaning of the probability

of particle dwelling in the spatial range (x_i, x_f) at the moment t (see also [67]. And the equivalency of relations (19) and (20) is a consequence of the continuity equation which connects the staying (dwelling) and the motion (traversing, transferring, passing, entering, outgoing) processes. However, we note that the applicability of the measure $P(x_1, x_2; t)$ for the time analysis (in contrast to the space analysis) is limited since it serves directly for calculations of only dwelling durations but not of their distributions.

Taking into account that $J(x_i, t) = J_{in}(x_i, t) + J_R(x_i, t) + J_{int}(x_i, t)$ and $J(x_f, t) = J_T(x_f, t)$ with J_{in}, J_R and J_T corresponding to the wave packets $\Psi_{in}(x_i, t), \Psi_R(x_i, t)$ and $\Psi_T(x_f, t)$, constructed from the stationary wave functions Ψ_{in}, $\Psi_R = A_R \exp(-ikx)$ and Ψ_T, respectively,

$$J_{int}(x, t) = \mathrm{Re}\{(i\hbar/m)[(\Psi_{in}(x, t))(\partial\Psi_R^*(x, t)/\partial x) + (\Psi_R(x, t)(\partial\Psi_{in}^*(x, t)/\partial x)]\}$$

and

$$\int_{-\infty}^{\infty} J_{int}(x_i, t)\mathrm{d}t = 0,$$

we obtain

$$< \tau^{dw}(x_i, x_f) >=< T >_E< \tau_T(x_i, x_f) > + < R(x_i) >_E< \tau_R(x_i, x_i) > \tag{21}$$

with

$$< T >_E =< |A_T|^2 v >_E \Big/ < v >_E ; < R(x_i) >_E =< R >_E + < r(x_i) >;$$
$$< R >_E =< |A_R|^2 v >_E \Big/ < v >_E , < R >_E + < T >_E = 1$$

and

$$< r(x) >= \int_{-\infty}^{\infty} [J_+(x, t) - J_{in}(x, t)]\mathrm{d}t \Big/ \int_{-\infty}^{\infty} J_{in}(x, t)\mathrm{d}t.$$

We stress that $< r(x) >$ is negative and tends to 0 when x tends to $-\infty$.

When $\Psi_{in}(x_i, t)$ and $\Psi_R(x_i, t)$ are well separated in time, i.e., $< r(x_i) >= 0$, we obtain the simple weighted average rule

$$< \tau^{dw}(x_i, x_f) >=< T >_E< \tau_T(x_i, x_f) > + < R >_E< \tau_R(x_i, x_i) > . \qquad (22)$$

For a rectangular barrier with $\chi a \gg 1$ and quasi–monochromatic particles, the expressions (21) and (22) with $x_i = 0$ and $x_f = a$ pass to the known expressions

$$< \tau^{dw}(x_i, x_f) >=< \hbar k/\chi V_0 >_E \qquad (23)$$

(taking account of the interference term $< r(x_i) >$) and

$$< \tau_{dw}(x_i, x_f) >=< 2/\chi v >_E \qquad (24)$$

(when the interference term $< r(x_i) >$ is equal to 0). When $A_R = 0$, i.e., the barrier is transparent, the mean dwell time (19), (20) is automatically equal to

$$< \tau^{dw}(x_i, x_f) >=< \tau_T(x_i, x_f) > . \qquad (25)$$

It is not clear how to define *directly* the variance of the dwell–time distribution. The approach, proposed in [68], is rather sophisticated, with an artificial abrupt switching on the initial wave packet. It is possible to define the variance of the dwell–time distribution indirectly, in particular, by means of relation (21), basing on the variances of the transmission–time and reflection–time distributions, or by means of relation (19), basing on the variances of the positions x_1 and x_2.

IV. ANALYSIS OF THE LARMOR AND BÜTTIKER–LANDAUER CLOCKS.

One can often realize that introducing additional degrees of freedom as "clocks" distorts the true value of the tunnelling time. The Larmor clock uses the phenomenon of changing the spin orientation (The Larmor precession or spin–flip) in a weak homogeneous magnetic field covered the barrier region. If initially the particle spin is polarized in the x direction, after tunnelling the spin develops small y and z components. The Larmor time $\tau^{La}_{y,T}$ and $\tau^{La}_{z,T}$ are defined by the ratio of the spin–rotation angles around z–axis and y–axis (in turn defined by the developed $y-$ and z–*spin* components respectively) to the precession (rotation) frequency [9,10,61]. For an opaque rectangular barrier with $\chi a \gg 1$ the expressions

$$< \tau^{La}_{y,tun} >=< \tau^{dw}(x_i, x_f) >=< k/\chi V_0 >_E \tag{26}$$

and

$$< \tau^{La}_{z,tun} >=< ma/\hbar\chi >_E \tag{27}$$

were obtained.

In [35,64] it was noted that, if the magnetic field region is infinite, the expression (26) passes into the expression (14) for the phase tunnelling time, after averaging over the small energy spread of the wave packet.

As to (27), in the reality we have not a precession but a jump to position " spin–up" or "spin–down" (spin–flip) accompanyed by the Zeeman energy–level splitting [6,47]. Due to the Zeeman splitting, the component of the spin, that is parallel to the magnetic field, corresponds to a higher tunnelling energy and hence it tunnels preferentially. And, namely, therefore one can realize that this time is connected with the energy dependance of $|A_T|$ and coincides with the expression (15).

The work of the Büttiker– Landauer clock is connected with the modulation cycle (absorption or emission of modulation quanta), caused by the oscillating part of a barrier, during tunnelling. And also in this case, one can realize that the coincidence of the Büttiker–Landauer time with (15) is connected with the energy dependence of $|A_T|$ for the same reasons as for $< \tau^{La}_{z,tun} >$.

V. ANALYSIS OF THE MEAN TUNNELLING TIMES, DEFINED BY AVERAGING OVER KINEMATIC PATHS.

The Feynman path–integral approach to quantum mechanics was applied in [54–57] to study and calculate the mean tunnelling time averaged over all paths, that have the same beginning and end, with the complex weight factor $\exp[iS(x(t)/\hbar]$, where S is the action associated with the path $x(t)$. Namely such weighting of tunnelling times implies their distribution with a real and an imaginary components [36]. In [54] the real and imaginary

parts of the obtained complex tunnelling time were found to be equal to $\tau^{La}_{y,tun}$ and $-\tau^{La}_{z,tun}$ respectively.

An interesting development of this approach, the *instanton* version, is presented in [57]. The instanton–bounce path is a stationary point of the Euclidean action. The latter is obtained by the analytic continuation to imaginary time in the Feynman path–integrals containing the factor $\exp(iS/\hbar)$. This path obeys a classical equation of motion in the potential barrier with the sign reversed. In [57] the instanton bounces were considered as a real physical processes. The bounce duration was calculated in real time and was found to be in good agreement with the one evaluated by the phase–time method. The temporal density of bounces was estimated in imaginary time and the obtained result coincided with (13) for the square root of the distribution variance at the limit of the phase–time approximation. Here one can see a manifestation of the virtual equivalence of the Schrödinger representation and the Feynman path–integral approach to quantum mechanics.

Another definition of the tunnelling time is connected with the Wigner distribution paths [58,59]. The basic idea of this approach, finally formulated by Muga, Brouard and Sala, is that the distribution of the tunnelling times in the dynamical evolution of wave packets through barriers can be well approximated by a classical ensemble of particles with a certain distribution function, namely the Wigner function $f(x,p)$, so that the flux at position x can be separated into positive and negative components:

$$J(x) = J^{+}(x) + J^{-}(x) \tag{28}$$

with

$$J^{+}(x) = \int_0^\infty (p/m) f(x,p) \mathrm{d}p$$

and $J^{-} = J - J^{+}$. Then *formally* the same expressions (3), (5) and (7) for the transmission, tunnelling and penetration durations and so on, as in the O–R formalism, were obtained with the substitution of $J^{\pm}$ instead of our $J_{\pm}$. The dwell time decomposition in this approach takes the form

$$< \tau^{dw}(x_i,x_f) >=< T >_E< \tau_T(x_i,x_f) > + < R_M(x_i) >_E< \tau_R(x_i,x_i) > \tag{29}$$

with $R_M(x) = \int_0^\infty |J^-(x,t)|\mathrm{d}t$. Asymptotically $R_M(x)$ tends to $< R >_E$ and (29) takes *formally* the known form (22).

One more alternative is the stochastic method for wave packets [69]. It also leads to real times but its numerical implementation is not trivial [59].

In [60] the Bohm approach to quantum mechanics was used to choose a set of classical paths which do not cross. The Bohm formulation can provide, on the one hand, a strict equivalent to the Schrödinger equation, and on the other hand, a base for the nonstandard interpretation of quantum mechanics [36]. The obtained expression in [60] for the mean dwell time is not only positive definite but gives the unambiguous distinction between particles that are transmitted or reflected:

$$\tau^{dw}(x_i,x_f) = \int_0^\infty \mathrm{d}t \int_{x_1}^{x_2} |\Psi(x,t)|^2 \mathrm{d}x = T\tau_T(x_i,x_f) + R\tau_R(x_i,x_i) \tag{30}$$

with

$$\tau_T(x_i,x_f) = \int_0^\infty \mathrm{d}t \int_{x_1}^{x_2} |\Psi(x,t)|^2 \Theta(x-x_c)\mathrm{d}x/T, \tag{31}$$

$$\tau_R(x_i,x_f) = \int_0^\infty \mathrm{d}t \int_{x_1}^{x_2} |\Psi(x,t)|^2 \Theta(x_c-x)\mathrm{d}x/R, \tag{32}$$

where T and R are, here, the mean transmission and reflection probability respectively, the bifurcation line $x_c = x_c(t)$, separating transmitted and reflected trajectories, is defined by relation

$$T = \int_{-\infty}^{+\infty} |\Psi(x,t)|^2 \Theta(x-x_c)\mathrm{d}x. \tag{33}$$

Factually, in addition to the difference in the temporal integration in this and our formalism ($\int_0^\infty$ and $\int_{-\infty}^\infty$ respectively), sometimes essential, this approach gives one more alternative in separating the flux by the line x_c :

$$J(x,t) = [J(x,t)]_T + [J(x,t)]_R \tag{34}$$

with

$$[J(x,t)]_T = J(x,t)\Theta[x - x_c(t)],$$

$$[J(x,t)]_R = J(x,t)\Theta[x_c(t) - x].$$

VI. PECULIARITIES OF THE TUNNELLING EVOLUTION.

The results of calculations presented in [53], within the Olkhovsky–Recami formalism, show that: (i) at variance with [70], no plot for the mean penetration duration of our wave packets presents any interval with negative values, nor with a decreasing for increasing x; (ii) the mean tunnelling duration does not depend on the barrier width a (the Hartmann–Fletcher effect); (iii) the quantity $< \tau_{tun}(0,a) >$ decreases when the energy increases; (iv) the value $< \tau_{pen}(0,x) >$ rapidly increases for increasing x near $x = 0$ and tends to an almost saturation value near $x = a$.

In Fig.2 the dependences of the values of $< \tau_{tun}(0,a) >$ from a are presented for electronic wave packets and rectangular barriers with the same parameters as in [53] ($V_0 = 10$eV; mean electron energies $E = 2.5, 5, 7.5$eV with $\Delta k = 0.02\AA^{-1}$ (curves 1a, 2a, 3a respectively); energy $E = 5$eV with $\Delta k = 0.06\AA^{-1}$ (curves 4a, 5a respectively)). The curves, corresponding to different energies and k, merge practically into one curve, 6. And since the dependence of $< \tau^{ph}_{tun} >$ from a is very weak, the dependance of $< \tau_{tun}(0,a) >$ from a is defined mainly by the dependence of $< t_+(0) >$ from a (curves 1b–5b). All these calculations manifest the negative value of $< t_+(0) >$ from a (see also [71]). Such "a–causal" advance can be interpreted as a result of the superposition and interference of incoming and reflected waves: the reflected–wave packet extinguishes the back edge of the incoming–wave packet, and the larger is the barrier width the larger is the part of the back edge of the incoming–wave packet which is extinguished by the superimposing reflected–wave packet, up to the saturation, when the contribution of the reflected–wave packet becomes almost constant, independently from a.

Besides all $< t_+(0) >$ are negative and the values of $< \tau_{tun}(0,a) >$ are always positive and, moreover, larger than $< \tau_{tun}^{ph} >$, in accordance with (16). In connection with this , it is relevant to note that the example with a *classical* ensemble of two particles (one with a large above–barrier energy and the other with a small sub–barrier energy), presented in [72], contradicts to our results not only because that tunnelling is a pure quantum phenomenon without a direct classical limit but, first of all, because in [72] it is overlooked the fact that the values of $< t_+(0) >$ are negative (for our initial condition). The last calculations of Zaichenko [71] (for the same parameters) have shown that such advance is noticeable also before the barrier front (however, only near the barrier wall) and, moreover, the values of $< \tau_{pen}(x_i, x_f) >$ are negative for $x_i = -a/5$ and $x_f = 0, a/5, 2a/5$ and a little larger values of x_f inside the barrier. But this result is not a–casual because the causality conditions (see relations (47) and (48) in Sec.8) are fulfilled in this case.

VII. ABOUT THE ANALOGY BETWEEN NONRELATIVISTIC PARTICLE AND PHOTON (ELECTROMAGNETIC WAVE–PACKET) TUNNELLING.

The formal mathematical analogy between the time–dependent quantum equations for the motion of relativistic particles and the time–dependent equation for electromagnetic wave propagation was studied in [73,74]. Here, we shall deal with the comparison of the solutions of the time–dependent Schrödinger equation for nonrelativistic particles and of the time–dependent Helmholtz equation for electromagnetic waves, considering not only the formal mathematical analogy between them, but also such similarity of the probabilistic interpretation of the wave function for a particle and of a classical electromagnetic wave packet, (being according to [75] the “wave function for a single photon”) which is sufficient for the identical definition of mean time instants and durations (and distribution variances and so on) of propagation, collision, tunnelling,etc., processes for particles and photons [76].

Concretely, we consider a hollow narrowed rectangular waveguide like depicted in Fig.3 (with cross section $a \times b$ of the narrow part, $a < b$), which was employed for the ex-

periments with microwaves [39]. Inside it, the time–dependent wave equation for any of vector quantities $\overrightarrow{A}, \overrightarrow{E}, \overrightarrow{H}$, ($\overrightarrow{A}$ is the vector potential with the subsidiary gauge condition $div \overrightarrow{A} = 0$; $\overrightarrow{E} = -(1/c)\partial\overrightarrow{A}/\partial t$ is the electric field strength; $\overrightarrow{H} = rot \overrightarrow{A}$ is the magnetic field strength) is

$$\Delta\overrightarrow{A} - (1/c^2)\partial^2\overrightarrow{A}/\partial t^2 = 0. \tag{35}$$

As it is known, (see, for instance [77–79]), for boundary conditions

$$\begin{aligned} &E_y = 0 \quad \text{for } z = 0, \quad \text{and } z = a, \\ &E_z = 0 \quad \text{for } y = 0, \quad \text{and } y = b, \end{aligned} \tag{36}$$

the monochromatic solution of (35) can be represented as a superposition of following waves:

$$\begin{aligned} E_x &= 0, \\ E_y^{\pm} &= E_0 \sin(k_z z)\cos(k_y y)\exp[i(\omega t \pm \gamma x)], \\ E_z^{\pm} &= -E_0(k_y/k_z)\sin(k_y y)\cos(k_z z)\exp[i(\omega t \pm \gamma x)], \end{aligned} \tag{37}$$

(we have chosen for definiteness TE–waves) with $k_z^2 + k_y^2 + \gamma^2 = \omega^2/c^2 = (2\pi/\lambda)^2$; $k_z = m\pi/a; k_y = n\pi/b; m$ and n being integer numbers. So,

$$\begin{aligned} \gamma &= 2\pi[(1/\lambda)^2 - (1/\lambda_c)^2]^{1/2}, \\ (1/\lambda_c)^2 &= (m/2a)^2 + (n/2b)^2, \end{aligned} \tag{38}$$

where γ is real ($\gamma = \mathrm{Re}\gamma$) if $\lambda < \lambda_c$ and γ is imaginary ($\gamma = i\chi_{em}$) if $\lambda > \lambda_c$. Similar expressions for γ were obtained for TH–waves [39,78].

Generally a solution of (35) can be written as a wave packet constructed from monochromatic solutions (37), similarly to a solution of the time–dependent Schrödinger equation for nonrelativistic particles in the form of a wave packet constructed from monochromatic

terms. Moreover, in the primary–quantization representation, a probabilistic single–photon wave function is usually described by a wave packet for $\overrightarrow{A}$ [75,80], for example

$$\overrightarrow{A}(\overrightarrow{r},t) = \int\limits_{k_0>0} \frac{\mathrm{d}^3 \overrightarrow{k}}{k_0} \overrightarrow{\kappa}(\overrightarrow{k})\exp(i\overrightarrow{k}\overrightarrow{r} - ik_0 t) \tag{39}$$

in the case of the plane waves, where $\overrightarrow{r} = \{x,y,z\}$; $\overrightarrow{\kappa}(\overrightarrow{k}) = \sum_{i=1}^{2} \kappa_i(\overrightarrow{k})\overrightarrow{e}_i(\overrightarrow{k})$; $\overrightarrow{e}_i\overrightarrow{e}_j = \delta_{ij}$; $\overrightarrow{e}_i(\overrightarrow{k})\overrightarrow{k} = 0, i,j = 1,2$ (or y,z if $\overrightarrow{k}\overrightarrow{r} = k_x x$); $k_0 = \omega/c = \epsilon/\hbar c; k = |\overrightarrow{k}| = k_0$, and $\kappa_i(\overrightarrow{k})$ is the amplitude for the photon to have momentum $\overrightarrow{k}$ and i–polarization , and $|\kappa_i(\overrightarrow{k})|^2 \mathrm{d}^3 \overrightarrow{k}$ is then proportional to the probability that the photon has a momentum between $\overrightarrow{k}$ and $\overrightarrow{k} + \mathrm{d}\overrightarrow{k}$ in the polarization state $\overrightarrow{e}_i$. Though it is not possible to localize photon in the direction of its polarization, nevertheless, in a certain sense, for the one–dimensional propagation, it is possible to use the space–time probabilistic interpretation of (39) along x–axis (the propagation direction) [81]. Usually one uses not the probability density and the probability flux density with the corresponding continuity equation *directly* but the *energy density* s_0 and the *energy flux density* s_x (although, in general, they represent components of not a 4–dimensional vector but the energy–momentum tensor) with the corresponding continuity equation [75] which we write in the two–dimensional (*spatially*, one–dimensional) form:

$$\partial s_0/\partial t + \partial s_x/\partial x = 0 \tag{40}$$

where

$$s_0 = [\overrightarrow{E^*}\ \overrightarrow{E} + \overrightarrow{H^*}\ \overrightarrow{H}]/4\pi, \quad s_x = c[\overrightarrow{E^*}\ \overrightarrow{H}]_x/8\pi \tag{41}$$

and x–axis is directed along the motion direction (the mean momentum) of the wave packet (39). We stress that for the spatially one–dimensional propagation the energy–momentum tensor of the electromagnetic field reduces to the two–component quantity, scalar term s_0 and 1–dimensional vector term s_x, for which continuity equation (40) is *Lorentz–invariant*. Then, as a normalization condition, one choose the equality of the spatial integrals of s_0 and s_x to the mean photon energy and the mean photon momentum respectively or simply the

unit energy flux density s_x. With this, by passing the problem of the impossibility of the direct space probabilistic interpretation of (39), we can define *conventionally* the probability density

$$\rho_{em}\,\mathrm{d}x = S_0\,\mathrm{d}x \;\Big/ \int S_0\,\mathrm{d}x, \quad S_0 = \int s_0\,\mathrm{d}y\,\mathrm{d}z \tag{42}$$

of a photon to be found (localized) in the spatial interval $(x, x + \mathrm{d}x)$ along x–axis at the moment t, and the flux probability

$$J_{em}\,\mathrm{d}t = S_x\,\mathrm{d}t \;\Big/ \int S_x\,\mathrm{d}t, \quad S_x = \int s_x\,\mathrm{d}y\,\mathrm{d}z \tag{43}$$

of a photon to pass through point (plane) x in the time interval $(t, t+\mathrm{d}t)$, quite similarly to the probabilistic quantities for particles. The justification and convenience of such definitions are also supported by the coincidence of the wave–packet group velocity and the velocity of the energy transport which was established for electromagnetic waves (at least, in the case of usual plane waves) in [82]. Hence, (i) in a certain sense, for time analysis along the motion direction, the wave packet (39) is quite similar to a wave packet for nonrelativistic particles and, (ii) similarly to the conventional nonrelativistic quantum mechanics, one can define the mean time of photon (electromagnetic wave packet) passing through point x [81] :

$$< t(x) >= \int_{-\infty}^{\infty} tJ_{em,x}\,\mathrm{d}t = \int_{-\infty}^{\infty} tS_x(x,t)\,\mathrm{d}t \;\Big/ \int_{-\infty}^{\infty} S_x(x,t)\,\mathrm{d}t \tag{44}$$

(where for the natural boundary conditions, $\kappa_i(0) = \kappa_i(\infty) = 0$, in energy representation $(\epsilon = \hbar c k_0)$, we can use the same form of time operator as for particles in nonrelativistic quantum mechanics and hence verify the equivalence of calculations of $< t(x) >, Dt(x)$, etc., in both time and energy representations). Then, one can use the same interpretation for the propagation of electromagnetic wave packets (photons) in media and waveguides when collisions, reflections and tunnelling can take place. In particular, for waveguides ,like depicted in Fig.3, with boundary conditions (36) and, decreasing and increasing waves when $k_x = \gamma = i\chi_{em}$.

In the case of fluxes which change their signs with time, we introduce, following [35,52,53], quantities $J_{em,x,\pm} = J_{em,x}\Theta(\pm J_{em,x})$ with the same physical meaning as for particles. Therefore, expressions for mean values and variances of distributions of propagation, tunnelling, transmission, penetration, and reflection durations can be obtained in the same way as in the case of nonrelativistic quantum mechanics for particles (with the substitution of J by J_{em}). In the particular case of quasi–monochromatic wave packets, using the stationary–phase method under the same boundary and measurement conditions, as considered in Sec.2 for particles, we obtain the identical expression for the *phase tunnelling time*

$$\tau^{ph}_{tun,em} = 2/c\chi_{em} \ \text{ for } \ \chi_{em}L \gg 1. \tag{45}$$

From (45), we can see, that when $\chi_{em}L > 2$ the effective tunnelling velocity

$$v^{eff}_{tun} = L/\tau^{ph}_{tun,em} \tag{46}$$

is more than c, i.e., *superluminal.* This result agrees with the results of the microwave–tunnelling measurements presented in [39] (see also [40], where, moreover, the effective tunnelling velocity was identified with the group velocity of the final wave packet corresponding to a single photon).

VIII. A REMARK ON RESHAPING (RECONSTRUCTING) PHENOMENON.

The superluminal phenomena, observed in the experiments with tunnelling photons and evanescent electromagnetic waves [39–42], generated a lot of discussions on relativistic causality. And, in connection with this, also an interest for similar phenomena, observed for the electromagnetic pulse propagation in a dispersive medium [83–85], was revived. Already for long there was ascertained that the *wave–front velocity* of the electromagnetic pulse propagation, when pulses have a step–function envelope, cannot exceed the velocity of light c in vacuum [79,86]. There, the *signal velocity* was also defined as the velocity of the propagation of the pulse main part in a medium which was shown to be less than

the wave–front velocity. These conclusions were confirmed by various methods and in various processes, including tunnelling [74,87–91]. In [79] the distinctions between the above mentioned velocities and the *group velocity* were also analyzed.

One of the argued problems consists in the absence of a step–function form of forward edges for realistic wave packets [74,91]. In such cases the conclusions of [79] can seem to be inapplicable. Nevertheless an infinite but sufficiently rapidly decreasing forward edge of a pulse can be cut off, *with any desired degree of the accuracy*, (defined, for instance, by a sensitivity of registration devices or a chosen mathematical approximation), without an essential distortion of the pulse spectral expansion. This can give a possibility to apply the conclusions of [79,86]. But independently from these reasonings, one can search a principal understanding of cases with superluminal group velocities (or effective velocities, like (46)) *without* violations of special relativity or causality.

A possible way of such understanding can consist in explaining the superluminal phenomena during tunnelling on the base of a pulse *attenuated reshaping (or reconstructing)* discussed at the classical limit earlier by [83–85]. The later parts of an input pulse are preferentially attenuated in such a way that the output peak appears shifted toward earlier times, arising from the forward tail of the incident pulse in a *strictly causal manner* [40]. In particular, the following reasonable scheme is quite compatible with the usual idea of causality: if an overall pulse attenuation is very strong and, in the same case, during tunnelling, the leading edge of the pulse is less attenuated than the trailing edge, then, the time envelope of the out–coming final small flux can be totally *under* the temporal initial–flux envelope, which should pass through the same position if its motion were free in vacuum (see also the discussion in [92–94]). And, if the dependence of A_T from energy is much more weak than the dependence of the weight factor in an initial wave packet, the spectral expansion and hence the geometrical form of the transmitted wave packet will be practically undistorted in comparison with the spectral expansion and the form of the initial wave packet (reshaping). But if the dependence of A_T from energy is not weak, then the pulse form and width can be noticeably changed (reconstruction).

The proposed scheme can be considered as a possible *sufficient* (but *not necessary*) causality condition and one can try to formulate more general causality condition when the temporal envelope of the final flux can even go out of the temporal envelope of the initial pulse flowing through the same position. Really, one can assume that the wave–packet spectral expansion and then the shape and width of the final pulse remain the same as for the initial pulse (reshaping). And, if the dependence of A_T from energy is not weak, then the pulse form and width can be changed (reconstructing). For example, the following relation

$$\int_{-\infty}^{T} [J_{in}(x_f, t) - J_{f,+}(x_f, t)\, \mathrm{d}t \geq 0, \quad -\infty < T < \infty \tag{47}$$

is quite acceptable. It does simply signify that during any semi–confined (from above) time interval, an integral final flux (along any direction) does not exceed that integral flux which should pass through the same position during the free motion (with the light velocity c for photons) of the initial wave packet in vacuum, although, by the way, one can find such finite T_1 and T_2 , $(-\infty < T_1 < T_2 < \infty)$ for which

$$\int_{T_1}^{T_2} [J_{in}(x_f, t) - J_{f,+}(x_f, t)]\, \mathrm{d}t < 0.$$

One can also propose another causality condition:

$$\begin{aligned} &\int_{-\infty}^{T_0} t\ J_{f,+}(x_f, t)\, \mathrm{d}t \ \Big/ \int_{-\infty}^{T_0} J_{f,+}(x_f, t)\, \mathrm{d}t \\ &- \int_{-\infty}^{T_0} t J_{in}(x_f, t)\, \mathrm{d}t \ \Big/ \int_{-\infty}^{T_0} J_{in}(x_f, t)\, \mathrm{d}t \geq 0 \end{aligned} \tag{48}$$

where T_0 is the instant of the intersection of the temporal envelopes of both fluxes *after* the final–peak appearance. Relation (48) does simply signify that there is a *delay* in the averaged appearance of the forward part of the final wave packet, in comparison with the averaged appearance of the forward part of that wave packet, which should pass through the same position x_f, during the free motion of the initial wave packet in vacuum. The conditions (47) and (48) are much more general than the previous one. The same relations can be also used for the nonrelativistic causality conditions, with the only substitution of x_f by x_i in J_{in}.

It is curious that, without violating such causality, a certain undistorted *information*, carried out by a low–frequency modulation of a high–frequency wave packet, can be transmitted (however, with a strong attenuation) with a *superluminal* wave–packet group (or effective) velocity when attenuated reshaping takes place.

IX. CONCLUSIONS AND PROSPECTS

I. Now one can conclude that the basic quantum–physical formalism for determining the collision and tunnelling times for nonrelativistic particles and for photons has been already, at least in principle, constructed: (1) there are self–consistent definitions of mean time instants and time durations of various collision processes (including tunnelling) together with variances of their distributions, based on utilizing the properties of time as a *quantum–physical observable* (in quantum mechanics and in quantum electrodynamics), just similarly to other observables; (2) these definitions are functioning rather well, at least for *asymptotic* distances between initial wave packets and interaction regions and *finite* distances between final wave packets and interaction regions. In these cases, the phase–time, clock and instanton approaches give the results which are coincident with the mean duration or the square root of the duration–distribution variance obtained within our formalism. And, the asymptotic mean dwell time is the weighted average sum of the corresponding tunnelling and reflection durations. Moreover, such " asymptotic" coincidence can be naturally extended, if we take into account, also, the mean squared time duration

$$< [\tau_N(x_i, x_f)^2] >= [< \tau_N(x_i, x_f) >]^2 + D\tau_N(x_i, x_f) \tag{49}$$

with

$$D\tau_N(x_i, x_f) = Dt_n(x_f) + Dt_+(x_i)$$

where index N signifies T or *tun* or *pen* or R, and $n = +$ or $-$. Relation (49) can be rewritten also in the following equivalent forms:

$$< [\tau_N(x_i, x_f)]^2 >=< [t_n(x_f)- < t_+(x_i) >]^2 > +Dt_+(x_i)$$

$$=< [< t_n(x_f) > -t_+(x_i)]^2 > +Dt_n(x_i). \quad (50)$$

And, now we see that $[< \tau_T^{ph} >]^2 + D\tau_T^{ph}$, the squared *hybrid time* $[(\tau_{y,tun}^{La})^2 + (\tau_{z,tun}^{La})^2]^{1/2}$, introduced by Büttiker [61], and the squared absolute value of the complex tunnelling time, in the Feynman path–integration approach, are examples of mean squared durations and, all three are coincident in the cases of the infinite spatial extension of the magnetic field for the Büttiker hybrid time and of the instanton version pf the Feynman formalism. By the way, this formalism has been earlier applied and tested in the time analysis of nuclear and atomic collisions for which the boundary conditions are experimentally and theoretically assigned in the region, asymptotically distant from the interaction region, where the incident (before collision) and final (after collision) fluxes are well separated *in time* , without any superposition and interference. And, it has been supported (see in particular, [25,26] and references therein) by the results: (i) the validity of a correspondence principle between the time–energy QM commutation relation and the CM Poisson brackets; (ii) the validity of an Ehrenfest principle for the average time durations; (iii) the coincidence of the quasi classical limit of our own QM definitions for time durations (when such a limit exists; i.e., for above–barrier energies) with analogous well-known expressions of classical mechanics; (iv) the analysis of all other known theoretical approaches to the definition of collision durations, on the base of our formalism; (v) the analysis of experimental data on direct and indirect measurements of nuclear–reaction durations, at the range 10^{-21}–10^{-15} sec, and, in particular, the extraction from these data of informations on compound–nucleus level densities with the appropriate juxtaposition of the obtained informations with data of other experiments.

Let us stress that for complete extracting the time–duration values from experimental data on indirect measurements of nuclear–reaction durations, it is necessary to utilize not only the expressions for mean durations but also correct definitions of the duration variances and the higher–order central moments of the duration distribution [26] which is provided

by our formalism. At least, let us note that such a formalism also provided useful tools for resolving some long– standing problems related to the *time–energy uncertainty relation* [25,26].

II. For the applications of the presented formalism to such cases, when one intends to consider, not only asymptotic distances but also the region *inside and near* interactions, we have revised the notation of averaging weight (or integration measure) in time representation, utilizing two measure $J_{\pm}(x,t)$ for calculations of instant and duration mean values, distribution variances, for particle moving, passing, transferring and the measure $\mathrm{d}P(x,t)$ or $P(x_1, x_2; t)$ for limited calculations of only mean durations for particle staying, or dwelling. And, with these three measures, we can arrange all known approaches (but, of course, within the *conventional quantum mechanics*), including the mean dwell time, the Larmor–clock times and the times given by various versions of the Feynman path–integration approach, into an unique *consistent* and *non–contradictory scheme* on the base of our formalism <u>even</u> inside and near barriers.

By the way, $\mathrm{d}P(x,t)$ can be used as the adequate averaging measure for defining mean values and variances of position and distance distributions (see also [14]), together with $J_{\pm}(x,t)$ as the measures for the definitions of the mean traversed distances during finite time intervals, for constructing the basic quantum–mechanical formalism of the space analysis of collision and propagation processes.

III. The O–R flux separation scheme, within the conventional quantum mechanics (and quantum electrodynamics), is not the only possible one,although it is the only known *incoherent* flux separation without introducing any new postulates. Within the conventional quantum theory one can also get the physically clear (but mathematically not very suitable) *coherent* wave–packet separation by positive and negative momenta, which is explicit out of the barrier region and is obtainable by the momentum Fourier expansion inside the barrier region. This separation can be transformed in the incoherent flux separation, after utilizing the postulate of *quantum measurement theory*, about a possibility to describe measurement conditions by the corresponding projector, acting on wave functions, namely by

the projectors $\Lambda_{exp,\pm}$ onto positive–momentum and negative–momentum states respectively. There are also flux separation schemes within nonstandard versions of quantum theory (see examples in Sec.5). However, whatever separation scheme we choose, we have to keep to at least two necessary conditions: (1) the probabilistic meaning of every normalized flux component and, (2) passing to the standard flux expressions in the asymptotically remote spatial region, well–known in quantum collision theory (since the boundary conditions of *any* quantum collision in the asymptotic range are for long inspected and have not to depend from a chosen version of quantum mechanics).

For the space inside and near a barrier at least four kinds of separations of wave–packet fluxes do now coexist, with the fulfillment of these conditions for all of them. These separations are defined by different schemes, although univocally from the mathematical point of view: (i) the O–R separation $J = J_+ + J_-$ with $J_\pm = J\Theta(\pm J)$ was obtained within the conventional probabilistic continuity equation (following from the time–dependent Schrödinger equation) without any new physical postulate or any new mathematical approximation [53]. The asymptotic behaviour of the obtained expressions was tested by the comparison with other approaches and with the experimental results [35]; (ii) the proposed here separation $J = J_{exp,+} + J_{exp,-}$($J_{exp,\pm}$ being the fluxes which correspond to $\Lambda_{exp,\pm}\Psi(x,t)$ respectively) obtained also within the conventional probabilistic continuity equation, however after applying the projector (or wave–function *reduction*) postulate of quantum measurement theory. The asymptotic behaviour of the expressions, obtained on the base of this separation, is the same as in (i); (iii) relation (28) was obtained in the Muga–Brouard–Sala approach, according to the physical clear incoherent flux separation by positive and negative momenta, but with additional introducing the model of the Wigner–path distribution; (iv) relation (34) was obtained in the Leavens approach, according to the incoherent flux separation by trajectories of particles to be transmitted and to be reflected, with introducing the nonstandard Bohm interpretation of quantum mechanics.

The flux separation schemes (i), (iii), and (iv) give *asymmetric* expressions for the mean dwell time near a barrier (see the relations (21), (29), and (30)–(33) respectively), apparently

due to the right–left asymmetry of boundary conditions: we have a partially simultaneous coexistence of incident and reflected wave packets from the left and the only one transmitted wave packet on the right. The separation (ii) gives the *symmetric* expression (22) for the mean dwell time even near a barrier.

IV. From reasonings, presented in Sec.2, 6, and 8, one can easily see that positive values (or values inside (or on) the light cone for relativistic waves) of the collision or propagation or tunnelling duration is only a sufficient but not the necessary causality condition. Now, we have not a unique general formulation of the causality principle which would be necessary for all possible cases. In Sec.2 and Sec.8 some new formulations of the causality condition are proposed for possible approbations.

V. The phenomenon of reshaping (reconstructing), which was spoken about in Sec.8, as well as the advance phenomenon at the beginning of tunnelling, which was spoken about in Se.6, are closely connected with a coherent super–position of incoming and reflected waves. It is advisable to examine these phenomena from various viewpoints and within various approaches with the scope to elucidate *causality* condition during tunnelling. Moreover, the investigation of the phenomena of reshaping (reconstructing) and advance by themselves could pursue, moreover, two important purposes: it will inevitably be the necessary part of the future *kinematic* theory of the tunnelling of particles, waves, many–particle systems, and solitons inside and near potential barriers and besides, that can serve as a base of the birth and development of a new field of the physical information–*a superluminal* propagation of information (see also [91]).

VI. It is known that there are multiple internal reflections from the both potential walls and corresponding multiple penetrations through the walls during the particle motion inside a potential well or a potential barrier with above–barrier energies [95,96]. The sums of the multiply reflected and penetrated waves give the resulted reflected and transmitted waves with the final reflection and transmission amplitudes. Naturally, the following question arises: are there such multiple internal reflections and corresponding penetrations during particle tunnelling with sub–barrier energies ? In [96] this question was studied and replied

formally positively. But a simple analysis permits to clear up that the matching conditions give definite solutions only when inside a barrier the motion along the incident flux is described by the decreasing wave and the motion against the incident flux is described by the increasing wave. However, for such waves, the fluxes are always equal to zero. For resolving this paradox, one can try to analyze the momentum Fourier–expansions of decreasing and increasing waves. But then intricate paradoxes with a-causality appear. This problem is very curious and can born some surprises.

VII. As one can conclude, the O–R formalism, presented here, permits in principle to study temporal characteristics in the Schrödinger and Feynman representations (which are formally equivalent). By the way, an interesting attempt was undertaken in [97] to develop the self–consistent method for calculating time quantities related to the motion of a particle, utilizing the Feynman representation and comparing the proposed method with the O–R formalism (in its earlier version presented in [35], however without the separation $J = J_+ + J_-$).

VIII. There is one more possible formally equivalent to the Schrödinger and Feynman representation for examining the collision and tunnelling evolution. As it is known, in the quantum theory to energy E, two operators correspond– the operator $i\hbar\partial/\partial t$ and the hamiltonian operator $\mathcal{H}$ in terms of the coordinate and momentum operators. The duality of these operators is well seen from the Schrödinger equation $\mathcal{H}\Psi = i\hbar\partial\Psi/\partial t$. The similar duality takes place for time in quantum mechanics: besides the general form $-i\hbar\partial/\partial E$, which is valid for any physical system (in the continuum energy spectrum), it is possible to express the time operator $\mathcal{T}$, utilizing the commutation relation $[\mathcal{T}, \mathcal{H}] = i\hbar$, in terms of the coordinate and momentum operators too [21,26]. And, to study the collision and the tunnelling evolution, *via* the operator $\mathcal{T}$ with the corresponding equation $\mathcal{T}\Psi = t\Psi$, it can prove to be useful too, particularly for researching, the influence of the barrier form on the tunnelling time [26].

IX. Time analysis of more complex processes, such as formations and decays of metastable states and time correlations of fluctuations of various quantities in many–particle systems

with the accompaniment of tunnelling processes, has to be developed on the base of the adequate formalisms for such processes, which are not developed yet and were only marked in [26] for simple approximations.

X. Time analysis in processes in the discrete energy spectra (for instance, for evolving wave packets composed from bound states inside two–well potentials with a barrier between wells) is quite different from time analysis of processes in the continuous energy spectra. For such processes, one can use the formalism based on the properties of the time operator in the discrete energy spectra [26,76], and, durations of transitions, (with non–zero fluxes), from one well to another, are defined by the Poincare‘ period $2\pi\hbar/d_{min}$, where d_{min} is the maximal common divisor of the level distances. The latter is defined, mainly, by the minimal level splitting caused by the barrier and hence depends on the barrier transition (penetration) probability at the appropriate energies.

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FIGURE CAPTIONS

Fig. 1 - The incoming, reflected and transmitted plane waves in the stationary picture of a particle, colliding with a barrier and tunnelling through it.

Fig. 2 - Plots of $< t_+(a) >$ and $< t_+(0) >$ as functions of the variable a for different values of E and Δk of Gaussian wave packets.

Fig. 3 - The rectangular waveguide with narrow–part section of dimension b and length L.